Technical Notes

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An Adjustable Spring Rate Suspension System

T. E. Shoup*
University of Houston, Houston, Texas
and
G. E. Simmonds†
Rutgers University, New Brunswick, N.J.

ONE of the most challenging tasks in aerospace design occurs in the design of suspension systems subjected to both shock and vibration. The usual isolation method for decreasing the transmission of structural borne energy from one system to another is to interpose a relatively flexible member between the two structures. In the linear spring domain, the isolation of steady-state vibration is accomplished by means of a "soft" suspension having low damping. The isolation of transient shocks is accomplished by means of "hard" springs and moderate damping. Unfortunately, linear design solutions that are good for shock isolation are poor for vibration isolation and vice versa. Thus the design process encompasses a tradeoff for a best compromise in performance. It has been shown[1] that nonlinear elastic suspensions that stiffen as they deflect can incorporate both the advantages of "soft" and "hard" systems to achieve both transient and steady-state isolation. Unfortunately, the variety of simple, nonlinear suspension devices is somewhat limited. In a previous Note[2] a special suspension spring known as the "elastica" suspension was presented. Because of its simple construction and its symmetrical, nonlinear behavior, this device was found to have much promise for aerospace and transportation applications. It is the purpose of this Note to add to the versatility of the "elastica" suspension by demonstrating how the device can be adjusted to provide an enhanced selection of nonlinear spring rates.

The adjustable version of the "elastica" suspension is shown in Fig. 1. This fundamental suspension module is constructed from a matched pair of thin flexible strips each clamped to a fixed mounting block on one end and clamped to a common movable platform in the center. For this device, the length of each flexible strip is represented by L, the flexural rigidity of each strip is EI and the relative position of the two fixed ends is specified by the distance D. In the previous study, the value of D was fixed at $4L/\pi$. If this distance can be adjusted to some new fixed value, the increased relative spacing of the two fixed ends can be used to stiffen the overall load-deflection behavior. As the center platform is subjected to an applied load P it will displace a distance X from the equilibrium position. The maximum allowable deflection of the center platform will be determined by the parameter D. Clearly, the platform cannot move past the end blocks nor can the value of D exceed $2L$. Thus: $0 \leq D \leq 2L$ and the allowable deflection of the center platform will be restricted to be: $0 \leq |X| \leq |D/2|$.

Elastica curves can be described in terms of the exact expression for the moment-vs-curvature relationship for the bending of beams.[3] For example, a single, clamped strip of length L compressed by a load P_u so that its ends are W_u apart will assume the shape known as the undulating elastica described by

$$0 = 1 - 2k^2\cos^2(\theta)$$

and

$$0 = (W_u/L)[F(k,\tfrac{3\pi}{2}-\theta) - F(k,\tfrac{\pi}{2}-\theta)] - 2[E(k,\tfrac{3\pi}{2}-\theta) - E(k,\tfrac{\pi}{2}-\theta) - F(k,\tfrac{3\pi}{2}-\theta) - F(k,\tfrac{\pi}{2}-\theta)]$$

In these two equations $F(k,\phi)$ and $E(k,\phi)$ are elliptic integrals of the first and second type with modulus k and amplitude ϕ. If the deflection W_u is known, these equations can be solved for the two unknowns k and θ. Once these values are known the load and bending moment can be found from

$$\frac{P_u L^2}{EI} = [F(k,\tfrac{\pi}{2}-\theta) - F(k,\tfrac{3\pi}{2}-\theta)]^2$$

and

$$\frac{ML}{EI} = 2k\sqrt{\frac{PL^2}{EI}}$$

A single clamped strip of length L stretched by a load P_n so that its ends are W_n apart will assume the shape known as the nodal elastica described by

$$0 = k\left(\frac{P_n L^2}{EI}\right)^{1/2} + F(\tfrac{1}{k},\tfrac{\pi}{4}) - F(\tfrac{1}{k},\tfrac{3\pi}{4})$$

and

$$0 = (W_n/L)\left(\frac{P_n L^2}{EI}\right|^{1/2} - 2k[E(\tfrac{1}{k},\tfrac{3\pi}{4}) - E(\tfrac{1}{k},\tfrac{\pi}{4}) + F(\tfrac{1}{k},\tfrac{\pi}{4}) - F(\tfrac{1}{k},\tfrac{3\pi}{4})] + \frac{1}{k}[F(\tfrac{1}{k},\tfrac{3\pi}{4}) - F(\tfrac{1}{k},\tfrac{\pi}{4})]$$

If the deflection W_n is known, these equations can be solved for k and (P_nL^2/EI). The maximum bending moment can be found from

$$\frac{ML}{EI} = 2\left(\frac{P_n L^2}{EI}\right)^{1/2}(k^2 - 0.5)^{1/2}$$

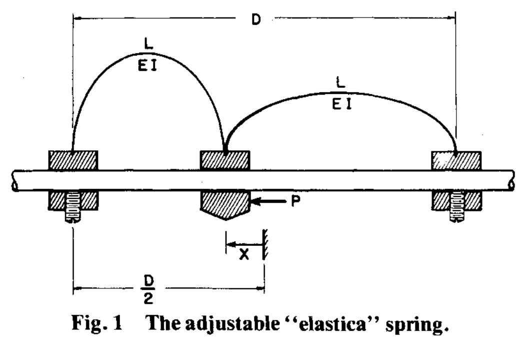

Fig. 1 The adjustable "elastica" spring.

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*Associate Professor of Mechanical Engineering.

†Graduate Student of Mechanical, Industrial, and Aerospace Engineering.

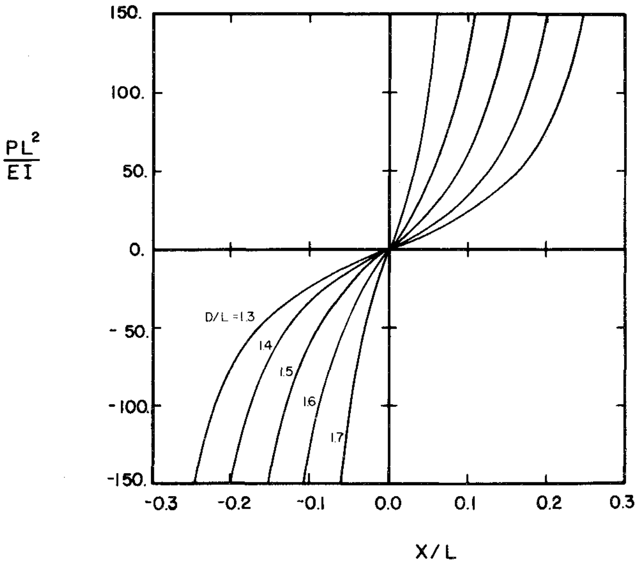

Fig. 2 Nondimensional load-deflection behavior for the adjustable "elastica" spring.

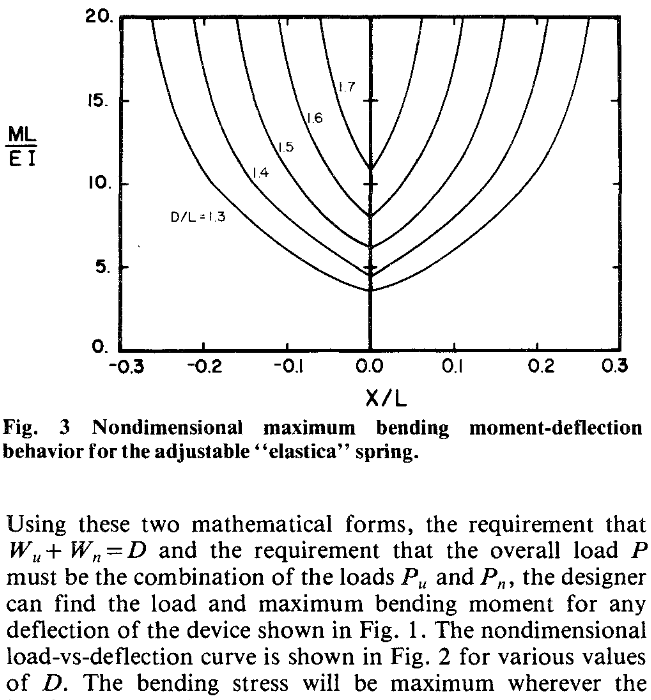

Fig. 3 Nondimensional maximum bending moment-deflection behavior for the adjustable "elastica" spring.

Using these two mathematical forms, the requirement that $W_u + W_n = D$ and the requirement that the overall load P must be the combination of the loads P_u and P_n, the designer can find the load and maximum bending moment for any deflection of the device shown in Fig. 1. The nondimensional load-vs-deflection curve is shown in Fig. 2 for various values of D. The bending stress will be maximum wherever the bending moment is maximum. This critical value will occur at the point where the curvature of the strip is maximum. For the design process, the value of M can be determined from the nondimensional plot shown in Fig. 3. Since the strips are assumed to be initially straight before they are installed, the bending moment does not equal zero when the suspension deflection is zero.

Once the parameter D has been selected, the dynamic analysis can be accomplished by traditional techniques.[4] When properly applied, the elastica suspension allows an initial "soft" support to minimize steady-state vibration transmission but will also stiffen at large deflections to provide "hard" behavior in the presence of shock loads. The selection of the parameter D allows the designer to choose the degree of initial stiffness necessary for a particular application.

Acknowledgment

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Motion of Spinning Spacecraft with Hinged Appendages

R. Sellappan* and P.M. Bainum†
Howard University, Washington, D.C.

Introduction

THE motion and stability of a spin-stabilized spacecraft with hinged appendages are treated. The dynamics of this type of fixed length appendage system during the deployment maneuver has been previously studied only for the case where the transverse components of the angular velocity vector are assumed to be zero throughout deployment and where the hinge points are located on the hub's principal transverse axes.[1] The present study considers the three-dimensional dynamics of a spin-stabilized spacecraft with hinged appendages where there is no restriction on the location of the hinge points. The motion and stability of such a system will be studied, analytically for special cases, and numerically for the general case.

Analysis

A. Equations of Motion

The hinged system to be studied is shown schematically in Fig. 1. The equations of motion in the five variables: ω_1, ω_2, ω_3, α_1, and α_2 are developed using the quasi-Lagrangian formulation for ω_i, $i=1,2,3$, and the general Lagrangian formulation for the variables α_i, $i=1,2$. The equations of motion for this system, neglecting external torques, are obtained as follows $[m/(M+2m) << 1]^2$:

$$
\begin{aligned}
& I_1\dot{\omega}_1-(I_2-I_3)\omega_2\omega_3+m[2(r_0^2+a^2+\ell^2)+2\ell\{r_0(s\alpha_1+s\alpha_2) \\
& \quad -a(c\alpha_1+c\alpha_2)\}]\dot{\omega}_1-m\{2a^2-2a\ell(c\alpha_1+c\alpha_2) \\
& \quad +\ell^2(c2\alpha_1+c2\alpha_2)-2r_0^2-2r_0\ell(s\alpha_1+s\alpha_2)\}\omega_2\omega_3 \\
& \quad +2m\ell\{r_0(c\alpha_1\dot{\alpha}_1+c\alpha_2\dot{\alpha}_2)+a(s\alpha_1\dot{\alpha}_1+s\alpha_2\dot{\alpha}_2)\}\omega_1+m\ell\{a \\
& \quad \times(s\alpha_1-s\alpha_2)-r_0(c\alpha_1-c\alpha_2)-(\ell/2)(s2\alpha_1-s2\alpha_2)\} \\
& \quad +(\omega_3^2-\omega_2^2)+m\{\ell\{\ddot{\alpha}_1-\ddot{\alpha}_2)+(\dot{\alpha}_1)^2(r_0c\alpha_1+as\alpha_1) \\
& \quad +\ddot{\alpha}_1(r_0s\alpha_1-ac\alpha_1)-(\dot{\alpha}_2)^2(r_0c\alpha_2+as\alpha_2) \\
& \quad -\ddot{\alpha}_2(r_0s\alpha_2-ac\alpha_2)\}=0 \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
& I_2\dot{\omega}_2-(I_3-I_1)\omega_3\omega_1+m\{2a^2-2a\ell(c\alpha_1+c\alpha_2)+\ell^2(c^2\alpha_1 \\
& \quad +c^2\alpha_2)\}\dot{\omega}_2-m\ell\{a(s\alpha_1-s\alpha_2) \\
& \quad -r_0(c\alpha_1-c\alpha_2)-(\ell/2)(s2\alpha_1-s2\alpha_2)\}\dot{\omega}_3+m\ell\{a(s\alpha_1 \\
& \quad -s\alpha_2)-r_0(c\alpha_1-c\alpha_2)-(\ell/2)(s2\alpha_1-s2\alpha_2)\}\omega_1\omega_2
\end{aligned}
$$

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*Senior Graduate Research Assistant, Dept. of Mechanical Engineering. Student Member AIAA.

†Professor of Aerospace Engineering, Dept. of Mechanical Engineering. Associate Fellow AIAA.